Lec 10 _ report 5

sec 3

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \qquad \xi = 0.5 \ , \ t_s = 2 \ sec$$

$$t_s = 2 = \frac{4}{\omega_n \xi} \Rightarrow \omega_n = \frac{4}{2 * 0.5} = 4$$

$$s_{1,2} = -\xi \omega_n \pm j \omega_n \sqrt{1-\xi^2} = -2 \pm j 2\sqrt{3}$$

$$K = \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix} M_c^{-1} \alpha_c(A)$$

$$M_c = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$$

$$\Rightarrow M_c^{-1} = -1 \begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$$

$$\alpha_c(s) = (s+2-j2\sqrt{3})(s+2+j2\sqrt{3}) = (s+2)^2 + 12$$

$$= s^2 + 4s + 16 = 0$$

$$\alpha_c(A) = A^2 + 4A + 16I = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}^2 + \begin{bmatrix} -4 & 0 \\ 4 & -8 \end{bmatrix} + \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 12 & 0 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 13 & 0 \\ 1 & 12 \end{bmatrix}$$

$$\Rightarrow K = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 13 & 0 \\ 1 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 38 & -12 \\ 25 & -12 \end{bmatrix} = \begin{bmatrix} 25 & -12 \end{bmatrix}$$